Batch Look Ahead Orthogonal Matching Pursuit

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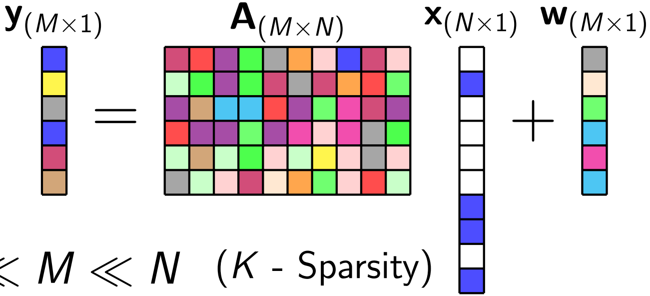
Sparse Signal Reconstruction



$$K \ll M \ll N \quad (K \text{ - Sparsity})$$

Sparse signal reconstruction refers to the problem of reconstructing a vector **x** from a set of incomplete and inaccurate measurements **y**, using a measurement matrix **A** (underdetermined system of equations since $M \ll N$). I.e., [1]

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{w} \tag{1}$$

Where **w** is the additive white noise which gets added in the process of sampling.

[1] Davenport M., Duarte M., Eldar Y., & Kutyniok G. (2012). Introduction to compressed sensing.

- The reconstruction of $\mathbf{x}$ from the knowledge of $\mathbf{y}$ and $\mathbf{A}$ is an ill posed problem as there are infinitely many possible $\mathbf{x}$'s which satisfy (1).
- However, by exploiting the sparsity of $\mathbf{x}$, the solution to the inverse problem of (1) can be thought of as the solution to the following optimization problem,

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\text{argmin}} \|\mathbf{x}\|_0 \text{ subject to } \mathbf{y} = \mathbf{A}\mathbf{x} \tag{2}$$

- In the presence of noise, the constraint on $\mathbf{x}$ can be relaxed and we get,

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\text{argmin}} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 \text{ subject to } \|\mathbf{x}\|_0 \leq K \tag{3}$$

[1] $\|\mathbf{x}\|_0$ denotes the L_0 norm of $\mathbf{x}$. Where $\|\mathbf{x}\|_0 = |\text{supp}(\mathbf{x})|$ equals the number of non zero components of $\mathbf{x}$.

Reconstructions Algorithms

- Solving (2) and (3) directly involves sweeping through all possible subsets of the index set of $\mathbf{x}$, which increases exponentially with N.
- There are many algorithms proposed to solve (2) and (3) efficiently. They are mainly classified into two types.
  - Convex Relaxation: L_1 norm minimization

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \|\mathbf{x}\|_1 \text{ subject to } \mathbf{y} = \mathbf{A}\mathbf{x}$$

    It is also called as Basis Pursuit (**BP**).
  - Greedy Pursuit: The index set is iteratively estimated over time. Generally faster than convex relaxation algorithms. E.g. OMP, LAOMP, CoSaMP, Subspace Pursuit, FACS.

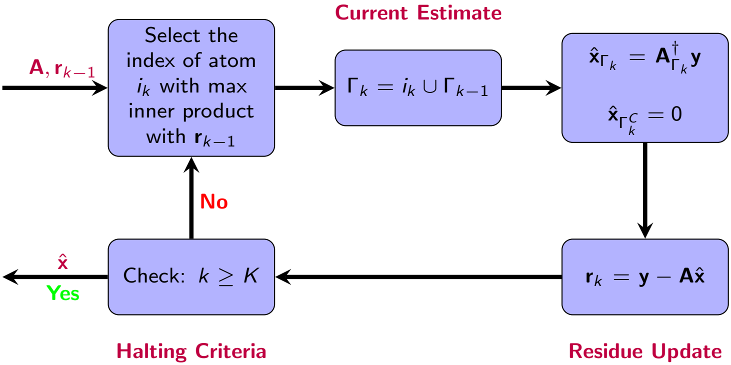


Figure: Block diagram representing the k^{th} iteration of OMP.

- **OMP** - A widely used greedy pursuit algorithm proposed by **J.A. Tropp and A.C. Gilbert**.
- **Drawback**: The atom giving the highest inner product may not always be a part of the original support set.

Look Ahead Orthogonal Matching Pursuit

- Proposed by **S. Chatterjee, D. Sundman and M. Skoglund**.
- Like OMP a single atom is chosen at the end of every iteration but differs in the way of choosing.
- L atoms (Ω) giving the maximum inner product with the current residue are chosen and individually given as inputs to a **Look Ahead Procedure**.
- **LAP** significantly improves the reconstruction accuracy over OMP. But this comes at the cost of an increased computation time.

Figure: Block diagram representing **Look Ahead Procedure** of LAOMP.

- The main drawback of LAOMP is that it is significantly more computationally complex than OMP.
- **Requirement**: Accuracy of LAOMP + speed of OMP.
- **Speed improvement**:
  - Selecting more than 1 atoms at the end of every iteration (CoSaMP, Susbspace Pursuit)
  - Running LAOMP on smaller subset of atom: RLAOMP
  - Reducing the number of calls to the **Look Ahead Procedure**
- With high probabilities the unused atoms again appear in Ω in subsequent iterations and sometimes only get added to Γ towards the very end.
- We choose the intersection of Ω with the index set predicted by the **LAP** (Λ_p) corresponding to the least norm residue.

Batch Look Ahead OMP

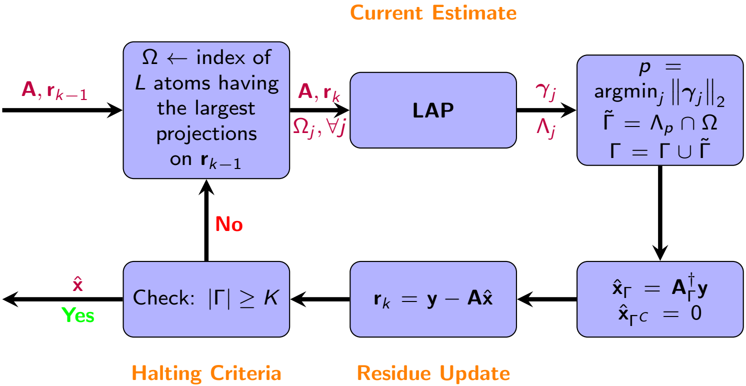


Figure: Block diagram representing the k^{th} iteration of BLAOMP-L.

- The look ahead parameter L is generally less than K.
- The number next to BLAOMP in the subsequent slides denote the value of L.

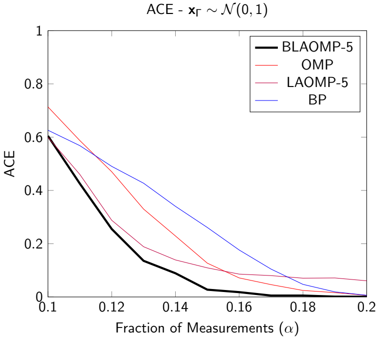
Numerical Experiment

The performance of the algorithms was analyzed using the following metrics,

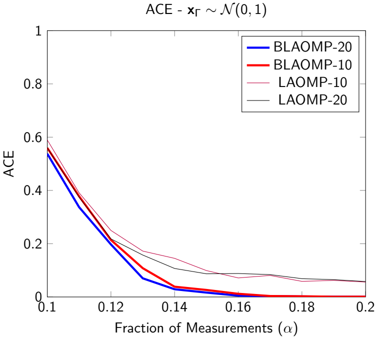
- **Average Cardinality Error** (**ACE**), $\mathbb{E}[CE]$, where $CE = d(\mathbf{x}, \hat{\mathbf{x}}) = 1 - \frac{|\Gamma - \hat{\Gamma}|}{K}$. Hence, $ACE = \frac{1}{T}\sum_{i=1}^{T} d(\mathbf{x}_i, \hat{\mathbf{x}}_i)$.
- **Average Signal to Reconstruction noise Ratio** (**ASRER**), $\mathbb{E}[SRER] = \frac{1}{T}\sum_{i=1}^{T} SRER_i$, where $SRER = \frac{\|\mathbf{x}\|_2}{\|\mathbf{x} - \hat{\mathbf{x}}\|_2}$
- **Computation time**, The time taken (in seconds) for the algorithm to find the reconstructed vector $\hat{\mathbf{x}}$ averaged over T iterations.

T - #iterations.

- $N = 500$ and $K = 20$, M is taken as a variable.
- For every M, matrix $\mathbf{A}$ is constructed such that $A_{ij} \sim \mathcal{N}(0,1)$ and the columns are normalized to have unit L_2 norm.
- For each $\mathbf{A}$, the vector $\mathbf{x}$ is generated 500 times with randomized support.
- The non-zero entries of $\mathbf{x}$ are sampled from,
  - Gaussian distribution, $\mathcal{N}(0,1)$.
  - From the set $\{-1,+1\}$.
- Fraction of Measurements, $\alpha = \frac{M}{N}$.
- The experiments are repeated 500 times and the values shown are averaged over 500 trials.



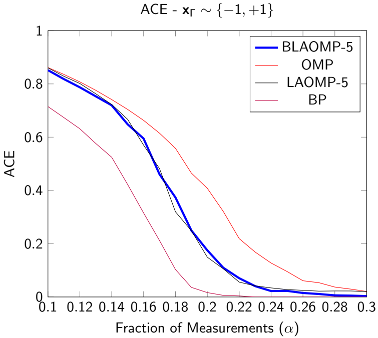
(a) ACE performance comparison between OMP, LAOMP, BP and BLAOMP.



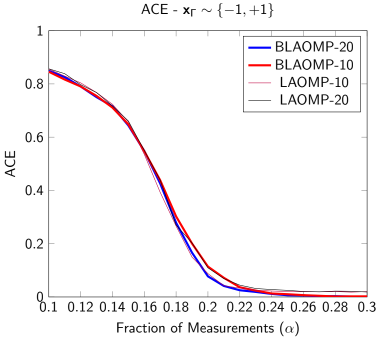
(b) ACE performance comparison between LAOMP and BLAOMP for different values of L.

Figure: ACE plotted vs fraction of measurements α when non-zero elements of $\mathbf{x}$ are sampled from Gaussian distribution, $\mathbf{x}_\Gamma \sim \mathcal{N}(0,1)$.

Performance of BLAOMP
Average Cardinality Error



(a) ACE performance comparison between OMP, LAOMP, BP and BLAOMP.

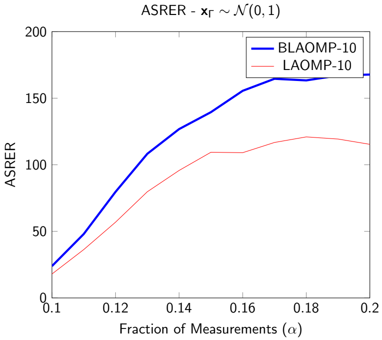


(b) ACE performance comparison between LAOMP and BLAOMP for different values of L.

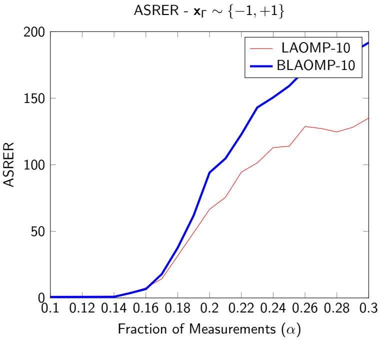
Figure: ACE plotted vs fraction of measurements α when non-zero elements of $\mathbf{x}$ are sampled from the set $\{-1,+1\}$, $\mathbf{x}_\Gamma \sim \{-1,+1\}$.

Performance of BLAOMP

Average Signal to Reconstruction noise Ratio



(a) ASRER performance comparison, $\mathbf{x}_\Gamma \sim \mathcal{N}(0,1)$.



(b) ASRER performance comparison, $\mathbf{x}_\Gamma \sim \{-1,+1\}$.

Figure: ASRER performance comparison between BLAOMP and LAOMP vs fraction of measurements α.

[1] SMNR = 41dB was taken for finding ASRER. SMNR $= \mathbb{E}(\|\mathbf{x}\|_2)/\mathbb{E}(\|\mathbf{w}\|_2)$

Algorithm	$\alpha = 0.15$	$\alpha = 0.16$	$\alpha = 0.17$	$\alpha = 0.18$	$\alpha = 0.19$	$\alpha = 0.20$
BLAOMP-20	**0.286**	**0.269**	**0.258**	**0.239**	**0.234**	**0.218**
BLAOMP-10	0.192	0.179	0.179	0.225	0.171	0.149
BLAOMP-5	**0.146**	0.137	0.136	0.128	0.119	0.115
LAOMP-20	1.487	1.402	1.464	1.491	1.510	1.495
LAOMP-10	0.651	7.489	0.758	0.722	0.754	0.713
LAOMP-5	0.356	0.350	0.368	0.380	0.380	0.370

Table: This table provides a comparison between the computation times of BLAOMP and LAOMP

The time values shown in the above table are computed using MATLAB R2015a on a Windows 8.1 computer having 8 GB of RAM and Intel Core i5 processor.

- Max improvement: $\frac{1.495}{0.218} \sim 6.85$.
- Min improvement: $\frac{0.356}{0.146} \sim 2.43$.

Conclusion

- The proposed algorithm (BLAOMP) removes the redundant iterations in LAOMP thereby reducing the convergence time.
- From the numerical experiments, Batch-LAOMP is able to achieve an improved performance over LAOMP for Gaussian sparse vectors, and similar performance to LAOMP for Rademacher sparse signal.

THANK YOU